

Risk & Reward



New Yorker 1979

Institute of Risk Management - Keele September 2004



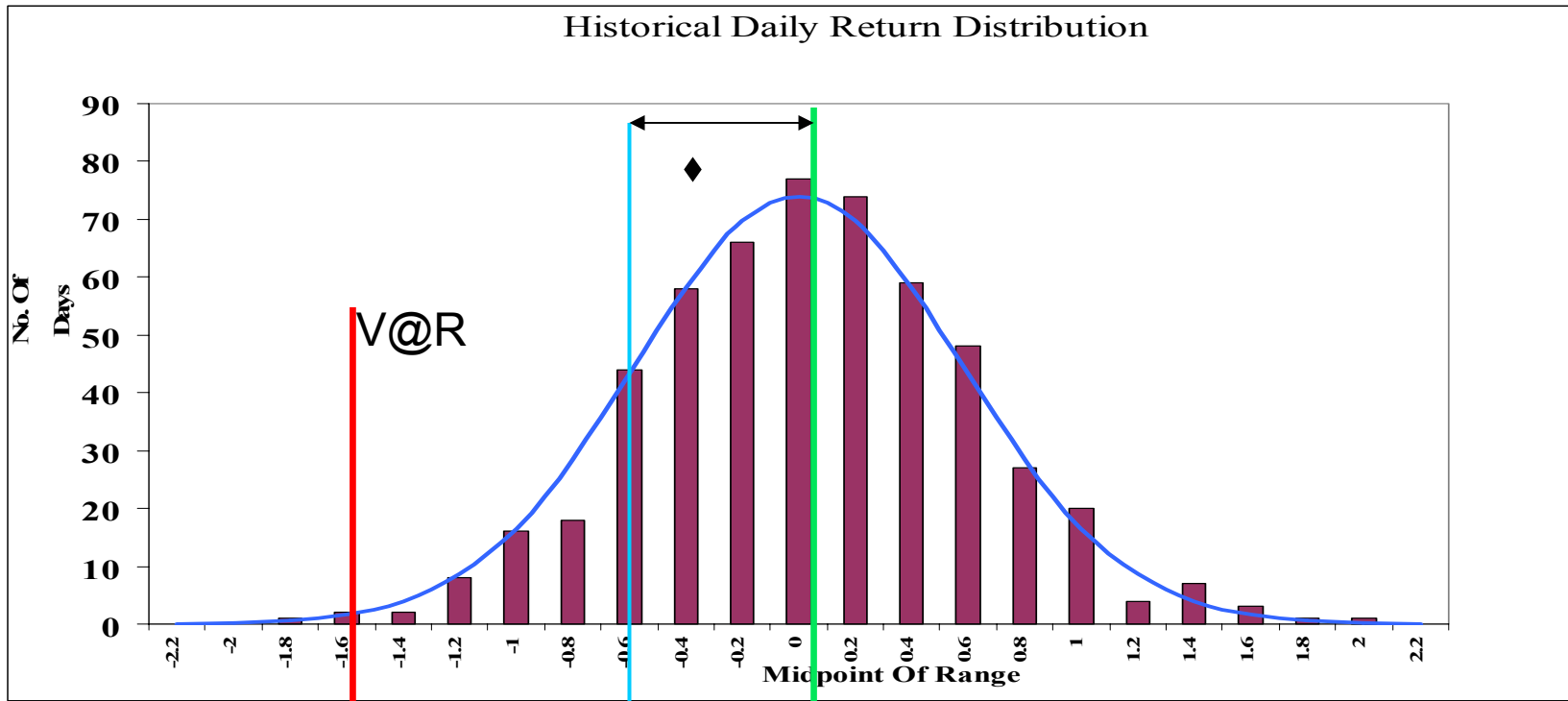
In the 25 years since...

- 1982 & 1993 Royal Society Papers
- ISO / IEC Guide 73 2002
- Risk Based Regulation in Finance
- Cooke Committee 1988 Basel I regulations
- NAIC RBC
- CAD and 1996 Model Amendments
- Value at Risk - of which more later.
- 2007 Basel II
- Operational Risk - of which more later.
- New Insurance Regulation
- Solvency II
- CP 190 CP 195 & FSA's PS 04/16



The Regulatory Approach

- Value at Risk - The maximum loss incurred with a certain probability.
- Tells us nothing about the losses in excess of the chosen probability.





Financial Regulation

- Value at Risk has been likened to driving a car using only the rear-view mirror for guidance.
- The new regulations add complexity to this for operational risk.
- Now we are driving the car using only the rear view mirror for guidance but we have to count and categorise the road-kill simultaneously.
- The Regulatory Panacea
- **Drive a more expensive car.**
- Capital Allocation and Risk Budgeting ??



Risk and Reward

- We are faced by uncertainty about the future
- In finance this uncertainty set is populated by returns
- Some we want
- Some we don't want
- The undesired elements of the uncertainty set are known as risk
- The desired outcomes are reward
- There are no natural partitions between risk and reward
- However Zero is a special point - more later
- We have an asymmetric preference for reward. We want the gains but not the losses.
- Asymmetry matters even in the case of "risk neutrality".
- We all need to compare distributions of returns to assess any investment.



The dimensions of Risk and Reward

- Likelihood
- Consequence
- The product of likelihood and consequence equal risk
- The Rumsfeld quotation:
- *There are the risks that we know that we know*
- *There are the risks that we know that we don't know*
- *And there are the risks that we don't know that we don't know.*
- Strictly the first of these is not a risk - it is an expectation and we price expectations.
- The second we provide capital against
- The third
- Scenario Analysis is an analysis of consequence alone.



The special nature of Zero

- This is not Siegel's Paradox - which reduces to the fact that $-20\% \oplus +20\%$.
- It is not the common frequently used approximation that $\log(1+x) \approx x$, which doesn't hold for large values of x .
- Nor is it the regrettable focus on the conditional distribution $F(x | x < 0)$
- It is not even simply the fact that probability distributions have a specific affine invariance - $[|a|. x + b]$
- **It is the fact that over strictly negative outcomes we prefer risk**
- We shall illustrate this point



Preference Reversal

- We are faced with an epidemic which will kill 600 people left unchecked.
- We have two available courses of medication
- The first will save two hundred lives with certainty
- The second will save 600 lives with probability $1/3$ and no lives with probability $2/3$
- Choose

- Large scale experiments show that we tend to choose the certainty.



Preference Reversal

- The same epidemic but now
 - We have two choices
 - The first course of treatment results in 400 deaths with certainty.
 - The second results in zero deaths with probability $1/3$ and 600 deaths with probability $2/3$.
 - Choose
-
- Large scale experiments show that we tend to choose the uncertain.



Preference Reversal

- It appears that we are treating deaths and lives as the negative equivalents of one another.
- It is also obvious however that we dislike uncertainty over positive outcomes
- But we like uncertainty over negative outcomes.
- This has serious ramifications for risk management
- Gambling for Resurrection
- Nick Leeson and a host of others



Risk, Return and Diversification

- Harry Markowitz in 1952 - Portfolio Optimisation
- The best mix of assets within a portfolio - maximisation of the risk return trade-off
- A simple mean- variance - correlation framework
- Chopra and Ziemba 1993 examined the effects of errors in these terms on the final portfolio.
- An error in the expected return (mean) was 10 times as important as an error in the risk (standard deviation) and that was four times as important as an error in the correlation.
- This suggests strongly that we should be concentrating our investment on return forecasting rather than risk assessment.



Diversification

- In the Value at Risk debate a set of axioms was developed by Artzner et Al. For “coherent” risk measures.
- One of these was the principle of sub-additivity
- That the risk of two events is no greater than their sum.
- This is also the principle of diversification of risk
- it is a result of the (partial) independence of the events.
- This is reasonable in the case of games against Nature
- If I carry an umbrella, it doesn't really affect the likelihood or severity of rainfall.
- But in the case of games against others...



Strategic Games

- With games against others, my behaviour influences other people's behaviour.
- We have the possibility of feed-back where risk may no longer be sub-additive.
- It may be super-additive
- Markets are mixed games!
- Diversification may achieve nothing!

- Decision paradoxes and Asymmetric preferences

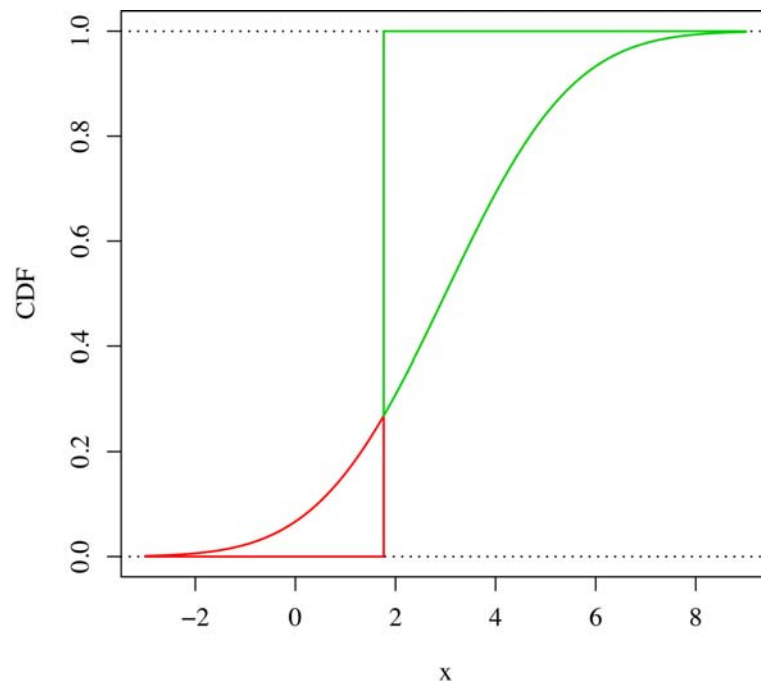
- Beware also of Cauchy distributed variables such as the pensions funding ratio.



Omega Functions

The Omega function of a distribution is an intuitive approach to **assessment of the quality of a bet**. It is defined by

$$\Omega_F(r) = \frac{\int_{-\infty}^{\infty} (1 - F(x)) dx}{\int_{-\infty}^r F(x) dx}$$

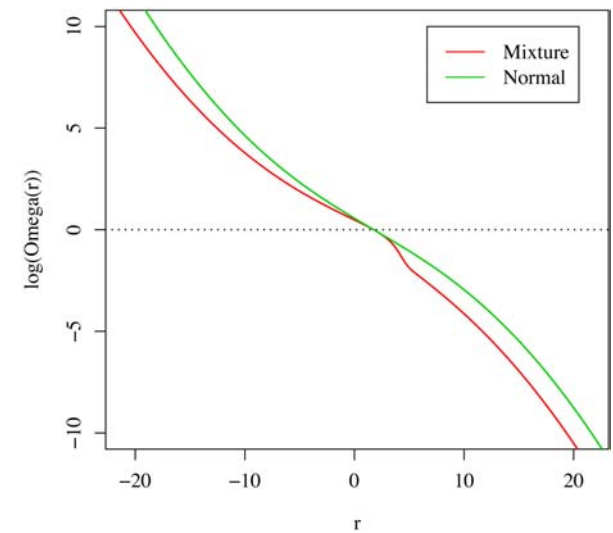
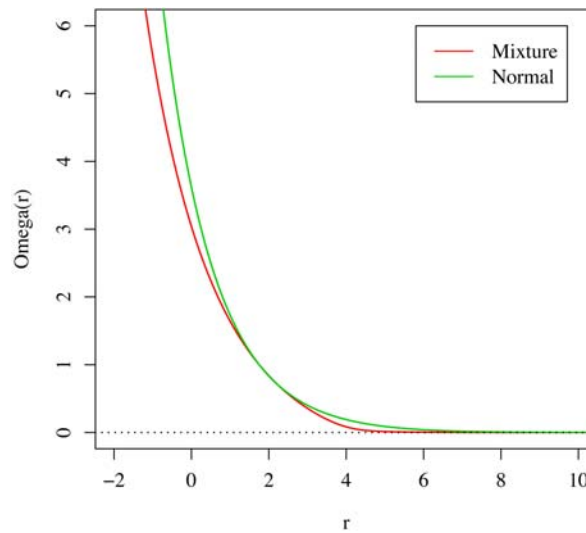
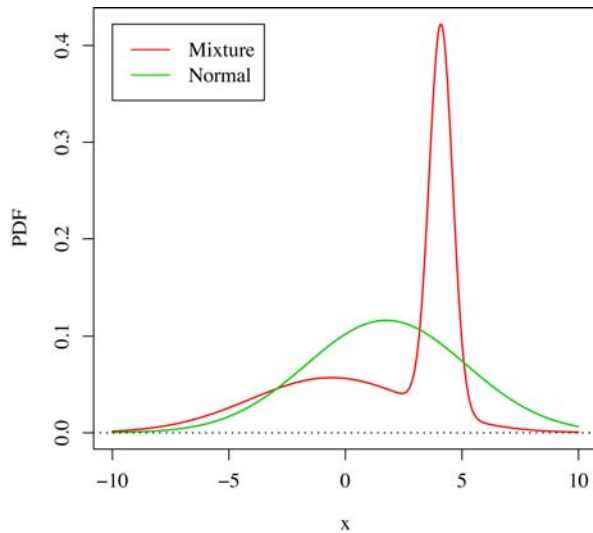


It corresponds to the sum of the amounts considered as a win multiplied by their corresponding probabilities divided by the sum of the amounts considered as a loss multiplied by their corresponding probabilities.



Characteristics of Omega Functions

- At any particular threshold r , the larger the Omega value the higher the quality of the bet.
- It is positive and monotone decreasing; it goes to infinity on the left and to zero on the right.
- It takes value 1 at the mean of the distribution.





Omega Functions Further

- The Omega function is mathematically **equivalent** to the cumulative distribution function (CDF). This means that
 - To each CDF corresponds a unique Omega function,
 - All information contained in the CDF is also contained in the Omega function (eg. all moments of the distribution, the tails behaviour, etc).

- Some equivalent formulations:

- Call-put ratio:

$$\Omega_F(r) = \frac{E_F \{ \max(x - r, 0) \}}{E_F \{ \max(r - x, 0) \}}$$

- A derivation of the put-call parity:

$$\Omega_F(r) = 1 + \frac{\mu_F - r}{\text{LPM}_{1,F}(r)}$$

- Partial moments formulation:

$$\Omega_F(r) = \frac{\text{UPM}_{1,F}(r)}{\text{LPM}_{1,F}(r)}$$



Lottery Paradoxes

Choose between two assets: “Buy” and “Sell”.

- Buy Payoff: A \$1 loss 999 times in 1000,
A \$999 gain 1 time in 1000.
- Sell Payoff: A \$1 gain 999 times in 1000,
A \$999 loss 1 time in 1000.

Sell’s distribution is the reflection about zero of Buy’s distribution.

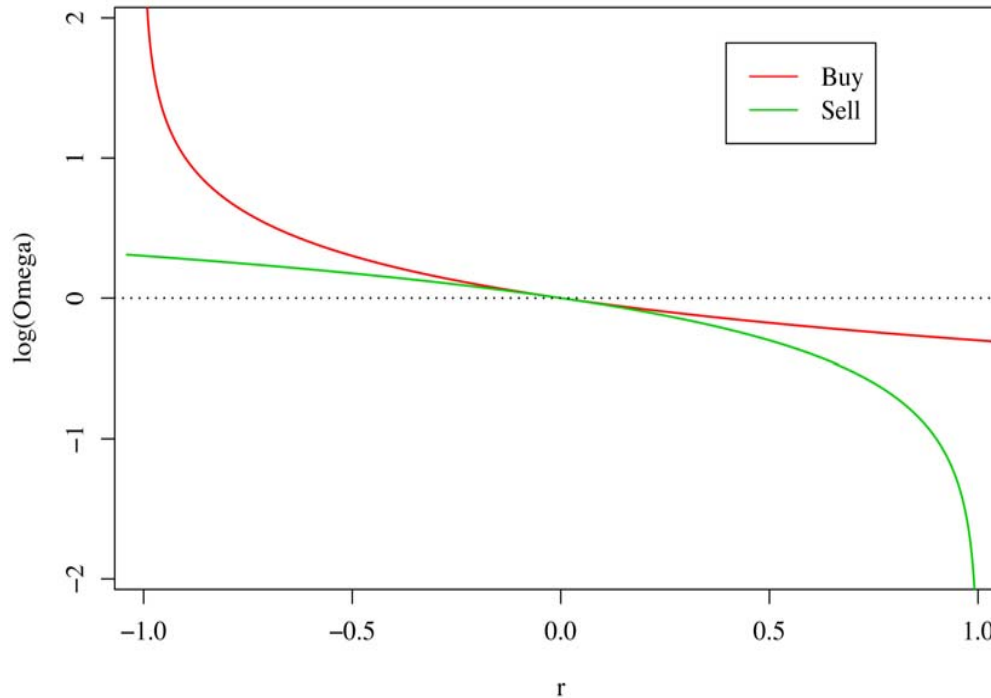
The Buy and Sell central moments:

- They have the same mean (zero) and variance (999),
- Skew is 31.57 for Buy and -31.57 for Sell,
- Kurtosis is 998 for Buy and Sell.



Omega functions of the Lottery

- The Buy Omega dominates the Sell Omega on the entire support, and therefore gets the correct allocation which is 100% Buy.
- This is generic for the lottery independent of mean or variance.

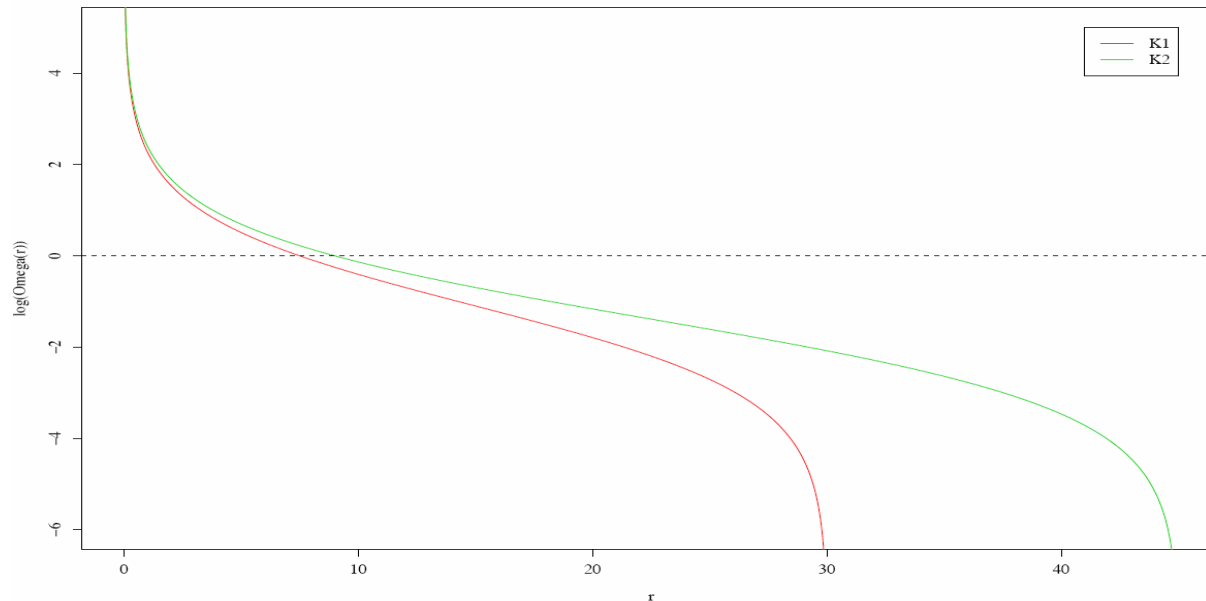




The Allais paradox

Here we are asked to make two choices - each between two alternates

The first choice is between a lottery, K1, which pays £30 with probability 25% and a lottery, K2, which pays £45 with probability 20% and otherwise zero.

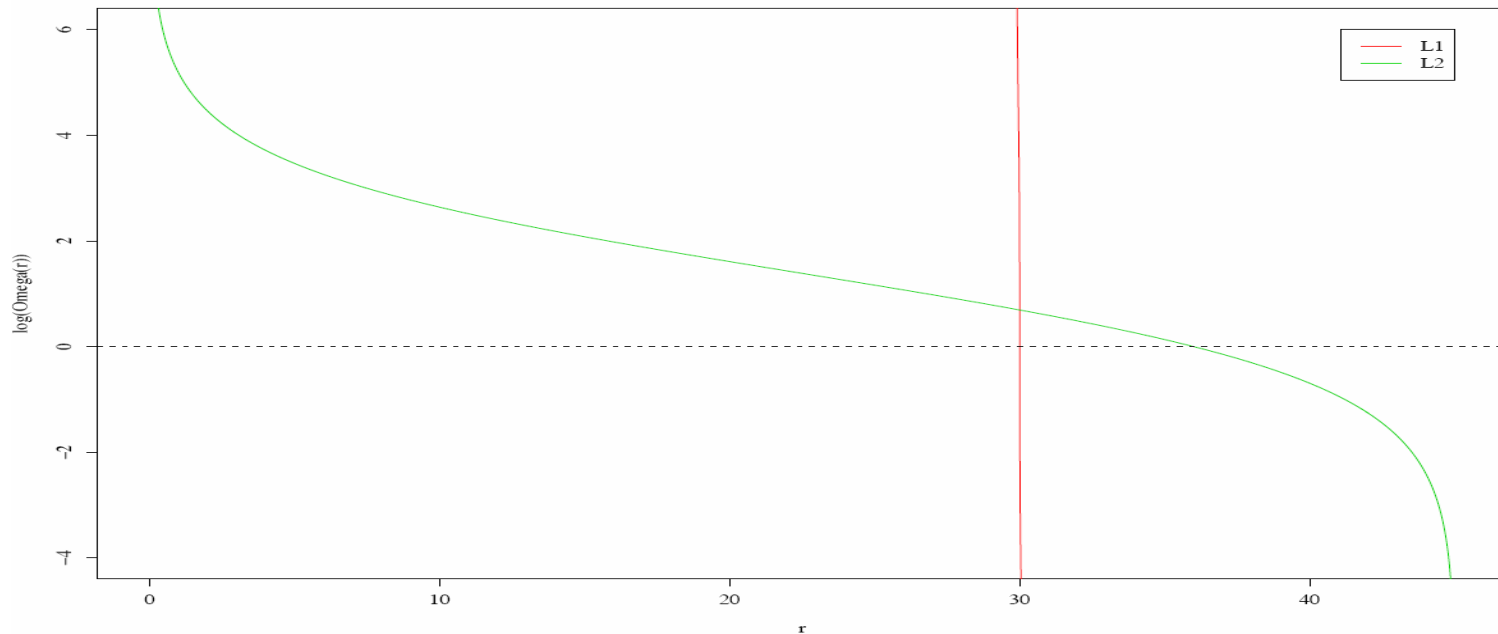


The Omega choice is immediate we prefer K2 to K1

This is in agreement with experimental findings.

The Allais Paradox (2)

Here we are offered a choice between £30 with certainty and a lottery which pays £45 with 80% probability and otherwise zero



Correcting for the asymmetry of the lottery alone (under risk neutrality), we find that we should accept any certainty of £ 29.25 or higher.

This accords with experimental observation.



Allais paradox

The process by which we extracted the certainty equivalent quoted earlier also tells us that there is limiting behaviour associated with asymmetric returns

Very low probability events can dramatically affect the re-evaluation of certainties

Diversification plays a major role in mitigating asymmetry effects



Conclusions

- Uncertainty and Risk are ever part of our lives
- The return side of the relationship often matters more than the risk
- The situation may be complex - markets are.
- Risk management is not just about diversification
- Mark Twain has a place.....and
- He advocated putting all our eggs in one basket and watching it like a hawk.
- Much of what we are concerned by is process and reliability engineering and that is the province of
- Operations Research
- The provision of Capital alone is no panacea.



Conclusions

- The precise detail of risk assessment and management can be critical.
- If my model is telling me surprising things, most likely **it is wrong.**
- Much of the risk aversion and revealed preference literature is suspect.
- **Common Sense is rather robust.**
- The risk-reward function, the Omega function, shows much promise as a decision aid.
- And has some truly beautiful mathematical properties.
- See WWW.FinanceDevelopmentCentre.com